Comment on "Electromagnetic convective cells in a nonuniform dusty plasma"

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Recently, Saleem and Haque [Phys. Rev. E **60**, 7612 (1999)] concluded that in the presence of a perturbed electron current parallel to an external magnetic field, the dispersion relation of the electrostatic convective cell and the magnetostatic modes is not modified. In the present Comment, the properties of electromagnetic as well as electrostatic waves in a nonuniform dusty magnetoplasma are reexamined, to demonstrate that Eq. (13) of the paper by Saleem and Haque as well as their conclusions are erroneous.

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Recently, Saleem and Haque [1] argued that finite parallel (to $\hat{\mathbf{z}}B_0$, where $\hat{\mathbf{z}}$ is the unit vector along the z axis and B_0 is the strength of the external magnetic field) wavelength effects to the dust-convective cell [2] can be introduced only by considering electromagnetic effects (viz., by incorporating the parallel component of the vector potential in the parallel wave electric field). This assertion is erroneous, as will be established in this Comment.

In the following, we reexamine the linear propagation of low-frequency (in comparison with the ion gyrofrequency), long-wavelength (in comparison with the ion gyroradius) electromagnetic waves in a nonuniform dusty magnetoplasma that contains equilibrium density gradients. For this purpose, we recapitulate the theory of Pokhotelov *et al.* [3], and discuss how the various dusty plasma modes are linearly coupled in a dusty magnetoplasma. We also demonstrate how it is possible to take the electrostatic limit from the general dispersion relation [3], so that one obtains the modification of the Shukla-Varma mode [2] due to the finite electron flow velocity perturbation parallel to the external magnetic field in a nonuniform dusty plasma.

Let us consider a multicomponent nonuniform dusty magnetoplasma whose constituents are electrons, singly charged ions, and negatively charged dust grains immersed in an external magnetic field $\hat{\mathbf{z}}B_0$. The dust grains are considered as point charges and their sizes, as well as the intergrain spacing, are assumed to be much smaller than the characteristic scale lengths (viz., the electron skin length, gyroradii, etc). The unperturbed plasma number density n_{j0} is assumed to be inhomogeneous along the x axis. The charged dust grains are considered to be immobile due to the very large dust grain mass. The quasineutrality condition at equilibrium is $n_{i0}(x) = n_{e0}(x) + Z_{d0}n_{d0}(x)$, where n_{j0} is the unperturbed number density of particle species j (j equals e for the electrons, i for the ions, and d for the negatively charged dust grains) and Z_{d0} is the number of charges residing on the dust grain surface.

In the low-frequency [in comparison with the ion gyrofrequency $\omega_{ci}(=eB_0/m_ic)$, where e is the magnitude of the electron charge, m_i is the ion mass, and c is the speed of light in vacuum], long-wavelength (in comparison with the ion gyroradius) electromagnetic fields, the electron and ion fluid velocities are

$$\mathbf{v}_e \approx \frac{c}{B_0}\hat{\mathbf{z}}\times\nabla_\perp\phi - \frac{cT_e}{eB_0n_{e0}}\hat{\mathbf{z}}\times\nabla_\perp n_{e1} + v_{ez}\hat{\mathbf{z}} \tag{1}$$

and

$$\mathbf{v}_i \approx \frac{c}{B_0}\hat{\mathbf{z}}\times\nabla_\perp\phi + \frac{cT_i}{eB_0n_{i0}}\hat{\mathbf{z}}\times\nabla n_{i1} - \frac{c}{B_0\omega_{ci}}(\partial_t + \mathbf{u}_{i*}\cdot\nabla)\nabla_\perp\phi, \tag{2}$$

where ϕ is the scalar wave potential, T_j is the temperature, n_{j1} ($\ll n_{j0}$) is the particle number density perturbation, and $\mathbf{u}_{i*} = (cT_i/eB_0n_{i0})\hat{\mathbf{z}}\times\nabla_\perp n_{i0}(x)$ is the unperturbed ion diamagnetic drift velocity. The parallel component of the electron fluid velocity is given by

$$v_{ez} \approx \frac{c}{4\pi n_{e0}e}\nabla_\perp^2 A_z, \tag{3}$$

where A_z is the parallel component of the vector potential. In the above, we have ignored the ion motion parallel to $\hat{\mathbf{z}}$, as well as neglected the compressional magnetic-field perturbation. Thus, the ion-acoustic and magnetosonic waves are decoupled in our low-β ($\beta \ll 1$) system.

Substituting Eq. (1) into the electron continuity equation, letting $n_j = n_{j0}(x) + n_{j1}$, and using Eq. (3), we obtain

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$$\partial_t n_{e1} - \frac{c}{B_0}\hat{\mathbf{z}}\times\nabla_\perp n_{e0}\cdot\nabla\phi + \frac{c}{4\pi e}\partial_z\nabla_\perp^2 A_z = 0. \quad (4)$$

On the other hand, substitution of the ion fluid velocity (2) into the ion continuity equation yields

$$\partial_t n_{i1} - \frac{c}{B_0}\hat{\mathbf{z}}\times\nabla_\perp n_{i0}\cdot\nabla_\perp\phi - \frac{c n_{i0}}{B_0\omega_{ci}}(\partial_t + \mathbf{u}_{i*}\cdot\nabla_\perp)\nabla_\perp^2\phi = 0. \quad (5)$$

Subtracting Eq. (5) from Eq. (4) and making use of Poisson's equation [viz., $\nabla^2\phi = 4\pi e(n_{e1}-n_{i1})$ for stationary dust grains], we obtain the modified ion vorticity equation

$$(\partial_t + u_{i0}\partial_y)\nabla_\perp^2\phi + \frac{\omega_{ci}^2}{\omega_{pi}^2}\partial_t\nabla^2\phi + \omega_{ci}\delta_d\kappa_d\partial_y\phi + \frac{v_A^2}{c}\partial_z\nabla_\perp^2 A_z = 0, \quad (6)$$

where $u_{i0} = (cT_i/eB_0 n_{i0})\partial n_{i0}/\partial x$ is the y component of the unperturbed ion diamagnetic drift velocity, $\omega_{pi} = (4\pi n_{i0}e^2/m_i)^{1/2}$ is the ion plasma frequency, $v_A = B_0/(4\pi n_{i0}m_i)^{1/2} \equiv c\omega_{ci}/\omega_{pi}$ is the Alfvén velocity, $\delta_d = Z_{d0}n_{d0}/n_{i0}$, and $\kappa_d = \partial \ln[Z_{d0}n_{d0}(x)]/\partial x$. The term $\omega_{ci}\delta_d\kappa_d\partial_y\phi$ is associated with the Shukla-Varma mode [2] in a nonuniform dusty magnetoplasma.

By using Eqs. (1) and (4), the parallel component of the electron momentum equation can be written as

$$(\partial_t + u_{e0}\partial_y)A_z - \lambda_e^2\partial_t\nabla_\perp^2 A_z + c\,\partial_z\left(\phi - \frac{T_e}{en_{e0}}n_{e1}\right) = 0, \quad (7)$$

where $u_{e0} = -(cT_e/eB_0 n_{e0})\partial n_{e0}(x)/\partial x$ is the y component of the unperturbed electron diamagnetic drift velocity, $\lambda_e = c/\omega_{pe}$ is the electron skin depth, $\omega_{pe} = (4\pi n_{e0}e^2/m_e)^{1/2}$ is the electron plasma frequency, and m_e is the electron mass. We note that in Eq. (7), in contrast to Ref. [1], we have also retained the parallel component of the electron pressure gradient force [viz., the last term on the left-hand side of Eq. (7)]. The latter is very important for low parallel phase velocity (in comparison with the electron thermal velocity) kinetic Alfvén waves for which we have to neglect the $\lambda_e^2\partial_t\nabla_\perp^2 A_z$ term.

Equations (4), (6), and (7) are the desired equations for the coupled drift-Alfvén-Shukla-Varma modes [3] in a nonuniform dusty magnetoplasma. The local dispersion relation can be derived by supposing that n_{e1}, ϕ, and A_z are proportional to $\exp(ik_y y + ik_z z - i\omega t)$, where $\mathbf{k} = k_y\hat{\mathbf{y}} + k_z\hat{\mathbf{z}}$ is the wave vector and ω is the frequency. Accordingly, in the local approximation, when the wavelength is much smaller than the scale length of the density gradient, we can Fourier-analyze Eqs. (4), (6), and (7) and combine them to obtain the general dispersion relation [3]

$$(\omega^2 - \omega\omega_m - \omega_{\rm IA}^2 k_y^2\rho_s^2)(\omega - \omega_{i*} - \omega_{\rm SV}) = \omega_{\rm IA}^2(\omega - \omega_{e*}), \quad (8)$$

where $\omega_m = \omega_{e*}/(1+k_y^2\lambda_e^2)$ is the magnetic drift wave frequency, $\omega_{j*} = k_y u_{j0}$, $\omega_{\rm IA} = k_z v_A/(1+k_y^2\lambda_e^2)^{1/2}$ is the frequency of the inertial Alfvén waves [4], $\omega_{\rm SV} = -\omega_{ci}\delta_d\kappa_d/k_y$ is the Shukla-Varma frequency [2] of the dust-convective cells in a nonuniform dusty magnetoplasma, $\rho_s = v_s/\omega_{ci}$ the ion Larmor radius at the electron temperature, and $v_s = (n_{i0}/n_{e0})^{1/2}(T_e/m_i)^{1/2} \equiv \delta c_s$ is the ion-acoustic velocity [5] in a dusty plasma. Here, $\delta = (n_{i0}/n_{e0})^{1/2} > 1$ and $c_s = (T_e/m_i)^{1/2}$. In deriving Eq. (8), we have assumed that $(k/k_y)^2(\omega_{ci}/\omega_{pi})^2 \ll 1$, where $k^2 = k_y^2 + k_z^2$.

We now examine Eq. (8) in various limiting cases. First, in a homogeneous dusty plasma, Eq. (8) correctly reproduces the frequency of the dispersive Alfvén waves [4], namely,

$$\omega = \omega_{\rm IA}(1 + k_y^2\rho_s^2)^{1/2}. \quad (9)$$

We note that for $k_y^2\lambda_e^2 \ll 1$, Eq. (9) gives $\omega = k_z v_A(1 + k_y^2\rho_s^2)^{1/2}$, which is the frequency of the kinetic (or the shear) Alfvén waves in an intermediate β ($m_e/m_i \ll \beta \ll 1$) plasma; the parallel (to $\hat{\mathbf{z}}$) phase velocity (ω/k_z) of the kinetic Alfvén waves is much smaller than the electron thermal velocity $v_{te} = (T_e/m_e)^{1/2}$. On the other hand, for $\omega/k_z \gg v_{te}$, we can neglect the $k_y^2\rho_s^2$ term in comparison with unity, and obtain from Eq. (9) $\omega = \omega_{\rm IA} \equiv k_z v_A/(1+k_y^2\lambda_e^2)^{1/2}$, which is the frequency of the dispersive inertial Alfvén waves in a very low-β plasma ($\beta \ll m_e/m_i$).

Second, for $\omega \gg \omega_m, \omega_{j*}$, we observe from Eq. (8) that the dispersive Alfvén waves are linearly coupled with the Shukla-Varma mode [2], $\omega = \omega_{\rm SV}$. Specifically, in a cold ($T_j \to 0$) dusty plasma with $\omega/k_z \gg v_{te}$ and $\omega \gg \omega_{i*}$, we obtain from Eq. (8)

$$\omega^2 - \omega\omega_{\rm SV} - \frac{k_z^2 v_A^2}{1 + k_y^2\lambda_e^2} = 0, \quad (10)$$

which clearly shows that the coupling between the Shukla-Varma mode [2] and the inertial Alfvén wave [4] arises due to the parallel electron motion in the wave electric and magnetic fields, which are $\mathbf{E} = -\nabla\phi - c^{-1}\partial_t A_z\hat{\mathbf{z}}$ and $\mathbf{B}_\perp = \nabla A_z \times \hat{\mathbf{z}}$, respectively. However, for $k_y^2\lambda_e^2 \gg 1$, Eq. (10) gives

$$\omega = \tfrac{1}{2}\,\omega_{\rm SV} \pm \tfrac{1}{2}\,(\omega_{\rm SV}^2 + 4\omega_{\rm cc}^2)^{1/2}, \quad (11)$$

where $\omega_{\rm cc} = (n_{e0}/n_{i0})^{1/2}(k_z/k_y)\omega_{\rm gm}$ is the frequency of the electrostatic convective cell (ECC) [6,7] in a dusty plasma, $\omega_{\rm gm} = (\omega_{ce}\omega_{ci})^{1/2}$ is the geometric mean frequency, and $\omega_{ce} = eB_0/m_e c$ is the electron gyrofrequency. Equation (11) explicity exhibits the modification of the Shukla-Varma frequency [2] ($\omega_{\rm SV}$) in the presence of the electrostatic wave field $\mathbf{E}$ ($= -\nabla\phi$), where the parallel (to $\hat{\mathbf{z}}$) *electrostatic field* ($E_z = -\partial_z\phi$) produces magnetic-field–aligned electron acceleration. We emphasize that in the limit $k_y^2\lambda_e^2 \gg 1$, the wave loses its electromagnetic characteristics. In fact, in the electrostatic limit, one must replace the last term on the left-hand side of Eq. (5) by $n_{e0}\partial_z v_{ez}$, and derive the corresponding modified ion vorticity equation (6) by replacing the last term on the left-hand side by $-(n_{e0}B_0\omega_{ci}/c)\partial_z v_{ez}$, with $\partial_t v_{ez} = (e/m_e)\partial_z\phi$, and Fourier-transform the resultant equations to derive Eq. (11). In a uniform dusty magnetoplasma, Eq. (11) yields $\omega=\omega_{\mathrm{cc}}$. The authors of Ref. [1] have failed to recognize that there is a parallel electron current $-n_{e0}ev_{ez}\equiv(n_{e0}e^2/m_e\omega)k_z\phi$ even in the electrostatic limit, and that this parallel electron current is associated with the ECC. Hence, contrary to the assertion of Ref. [1], the parallel wave-number modification of the Shukla-Varma mode [2] arises even in the electrostatic limit, and the statement of Ref. [1] on p. 7613 [after Eq. (13)] with regard to the role of the parallel electron current is fallacious because Eq. (13) of Ref. [1] is erroneous. Specifically, we observe that Eq. (13) of Ref. [1] is incompatible with

$$[(1+k_y^2\lambda_e^2)\omega-\omega_{e*}](\omega-\omega_{\mathrm{SV}})\omega=k_z^2v_A^2(\omega-\omega_{e*}), \tag{12}$$

which is obtained from Eq. (8) in the limits $k_y\rho_s\to 0$ (or vanishing parallel electron pressure gradient force) and $\omega_{i*}=0$; the latter have been assumed in Ref. [1] from the outset. Our Eq. (12) shows that the magnetostatic drift ($\omega=\omega_m$), the Shukla-Varma mode ($\omega=\omega_{\mathrm{SV}}$), the inertial Alfvén wave ($\omega=\omega_{\mathrm{IA}}$), and the electron drift wave ($\omega=\omega_{e*}$) are linearly coupled.

Third, when the parallel electron motion is completely neglected (viz., $k_zv_{ez}=0$), we see from Eq. (8) that flutelike magnetostatic [8] ($\omega=\omega_m$) and modified Shukla-Varma ($\omega=\omega_{i*}+\omega_{\mathrm{SV}}$) modes appear as independent normal modes of a nonuniform, dusty magnetoplasma with warm ions. Fourth, when the perpendicular wavelength is much larger than λ_e, we obtain from Eq. (8) for $\omega\gg\omega_{i*}$

$$(\omega^2-\omega\omega_{\mathrm{SV}}-k_z^2v_A^2)(\omega-\omega_{e*})=k_y^2\rho_s^2k_z^2v_A^2(\omega-\omega_{\mathrm{SV}}), \tag{13}$$

which exhibits the coupling between the drift-kinetic Alfvén waves and the Shukla-Varma mode due to finite Larmor radius correction of the ions at the electron temperature in a dusty plasma. Equation (13) resembles [9]

$$(\omega^2-\omega\omega_{i*}-k_z^2v_A^2)(\omega-\omega_{e*})=k_y^2\rho_s^2k_z^2v_A^2(\omega-\omega_{i*}), \tag{14}$$

which is the dispersion relation of the coupled drift-kinetic Alfvén waves in a warm electron-ion magnetoplasma without charged dust grains.

To summarize, we have reexamined the general dispersion relation (8) for low-frequency (in comparison with the ion gyrofrequency), long-wavelength (in comparison with the ion gyroradius) electromagnetic waves in a low-β nonuniform dusty magnetoplasma. We have stressed that the general dispersion relation (8) yields the frequencies of the well known electrostatic and electromagnetic waves in appropriate limits, contrary to the statements made in Ref. [1] with regard to the modification of the dispersion relation of the electrostatic convective cells and the magnetostatic modes due to a perturbed magnetic-field–aligned electron current. Furthermore, we have also identified an error in Eq. (13) of Ref. [1], which is incompatible with our Eq. (12), which follows from our general dispersion relation (8) when the parallel component of the electron pressure gradient force is ignored and when the ions are assumed to be cold, as supposed in Eqs. (3) and (4) of Ref. [1]. In conclusion, we stress that we have rectified the errors of Ref. [1] and have presented the correct description of dusty plasma waves in a nonuniform dusty magnetoplasma composed of warm electrons, warm ions, and immobile massive charged dust grains.

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